Numeric Response Questions

Progressions

Q.1 If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, their cubes respectively, then find the value of $\frac{S_3(1+8S_1)}{S_2^2}$.

Q.2 Let $S_v = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \cdots + \frac{1+2+\cdots+n}{1^3+2^3+\cdots+n^3}; n = 1,2,8, \ldots \ldots \ldots$ Then find maximum value of S_n.

Q.3 A square is drawn by joining the mid-points of the sides of a given square. A third square is drawn inside the second square in the same way and this process continuous indefiniting. If a side of the first square is 4 cm and the sum of the area of all the squares is α then find the value of $\alpha/4$.

Q.4 Lat ' p ' and ' q ' be the roots of the equation $x^2 - 2x + A = 0$, and Let " r " and " a " be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < x < s$ are in A.P. then find value of $(A + B)$.

Q.5 Find the number of terms common to the two sequences $17,21,25, \ldots ,417$ and $16,21,26, \ldots ,466$.

Q.6 The repeating decimal 0.429642964296... represents the fraction $\frac{\mathrm{m}}{3333}$ then find value of m.

Q.7 Between 1 and 31 are inserted m arithmetic means, so that the ratio of the 7^{th} and $(m-1)$ th means is $5:9$. Then find the value of m.

Q.8 If the ratio of sum of n terms of two different A.P's is $\frac{3n+5}{4n+3}$ then find the ratio of their 7^{th} terms.

Q.9 Find the maximum value of the sum of the A.P. $30,27,24,21, \ldots$

Q.10 If the sum of positive terms of the series $10 + 9\frac{4}{7} + 9\frac{1}{7} + \cdots \ldots$ is $\frac{k}{7}$ then find value of k.

Q.11 If $a_1, a_2, \ldots, a_{15}$ are in A.P. and $a_1 + a_5 + a_{15} = 15$, then $a_2 + a_3 + a_5 + a_{13} + a_{14} =$

Q.12 Let an be the nth term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{10} a_{2r-1} = \beta$, and the common difference of A.P. is $\frac{\alpha-\beta}{\lambda}$ then find λ

Q.13 Sum of first n positive terms of an A.P. is given by $\mathrm{S_n} = (1 + 2\,\mathrm{T_n})(1 - \mathrm{T_n})$. If the value of T_2^2 is $\frac{\sqrt{2}-1}{k\sqrt{2}}$ then find k

Q.14 If $\frac{3+5+7+\cdots\ldots+(2n-1)}{5+8+11+\cdots\ldots+10 \text{ terms}} = 7$ then find value of n.

Q.15 Find sum of the series $S = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \cdots$. upto 20 terms.

ANSWER KEY

1. 9.00	**2.** 2.00	**3.** 8.00	**4.** 74.00	**5.** 20.00	**6.** 1432.00	**7.** 14.00
8. 0.80	**9.** 165.00	**10.** 852.00	**11.** 25.00	**12.** 100.00	**13.** 2.00	**14.** 36.00
15. 115.00						

Hints & Solutions

1. $$\frac{S_3(1+8S_1)}{S_2^2} = \frac{\left[\frac{n(n+1)}{2}\right]^2\left(1+\frac{8n(n+1)}{2}\right)}{\left(\frac{n(n+1)(2n+1)}{6}\right)^2}$$

$$= \frac{9[1+4n(n+1)]}{(2n+1)^2} = 9$$

2. $$s_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \ldots + \frac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots\ldots+n^3};$$

$n = 1, 2, 3, \ldots\ldots$

we have $t_n = \frac{1+2+3+\ldots+n}{1^3+2^3+3^3+\ldots+n^3}$

$$= \frac{\Sigma n}{\Sigma n^3} = \frac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{\frac{n(n+1)}{2}}{\frac{n^2(n+1)^2}{4}}$$

$$= \frac{2}{n(n+1)} = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$\Rightarrow \sum_{k=1}^{n} t_k = 2\left\{\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n-1}\right)\right\}$$

$$= 2\left(1-\frac{1}{n+1}\right) = \frac{2n}{n+1}$$

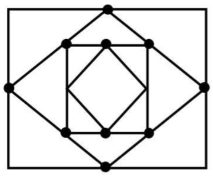
$$\Rightarrow s_n = \frac{2n}{n+1} = 2\left(1-\frac{1}{n+1}\right) = 2 - \frac{2}{n+1}$$

$$\Rightarrow s_n = 2 - \frac{2}{n+1} \Rightarrow s_n = 2 - \frac{2}{n+1} < 2$$

when $n \to \infty$, $s_n = 2$

$\therefore$ s_n is not greater than 2 Ans.

3. If a side of any square is x cm, then the side of the square obtained by joining its mid-points is given by



$$\sqrt{\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^2} = \frac{x}{\sqrt{2}} \text{ cm}$$

and such its area is

$$\left(\frac{x}{\sqrt{2}}\right)^2 = \frac{x^2}{2} \text{ cm}^2$$

Now the area of the first square is $4^2 = 16$ sq cm. The area of the second square is 859 sq cm, the area of the third square is 4 sq cm and so on. Hence the sum of the areas is given by

16 + 8 + 4 + 2 + … upto infinite

$$= \frac{16}{1-(1/2)}$$

$$= \frac{16}{1/2}$$

$= 16 \times 2$

$= 32$ cm² Ans.

4. Let common difference of A.P. is d. then

Let $p = \lambda$, $q = \lambda + d$, $r = \lambda + 2d$, $s = \lambda + 3d$

$\because$ $p + q = 2 \Rightarrow 2\lambda + d = 2$ ....(1)

and $r + s = 18 \Rightarrow 2\lambda + 5d = 18$ ....(2)

on solving (1) & (2) $\lambda = -1$ & $d = 4$

so $p = -1$, $q = 3$, $r = 7$, $s = 11$

5. First common term is 21

and common difference of series of common terms is LCM of 20.

So n^{th} term of series of common term is

$21 + (n - 1)\,20 = 20\,n + 1$

$\because \quad 20n + 1 \le 417$

So $n \le \frac{416}{20}$

$n \le 20.8$

So $n = 20$

6. $x = 0.429642964296$ ....(i)

$10000x = 4296.42964296$ ....(ii)

$9999x = 4296 \Rightarrow x = \frac{1432}{3333}$

7. $d = \frac{31-1}{m+1} = \frac{30}{m+1}$

$\frac{A_7}{A_{m-1}} = \frac{1+7d}{31-2d} = \frac{5}{9}$

$\Rightarrow d = 2 = \frac{30}{m+1} \Rightarrow m = 14$

8. $\frac{S_n}{S'_n} = \frac{3n+5}{4n+3}$

So for $\frac{T_7}{T'_7}$ put $n = (2 \times 7) - 1 = 13$

$\therefore \frac{T_7}{T'_7} = \frac{3(13)+5}{4(13)+3} = \frac{44}{55} = \frac{4}{5}$

9. Let $I_n = 0$

$30 + (n - 1)(-3) = 0$

$n - 1 = 10$

$n = 11$

Sum is max if $n = 11$

Max sum $= \frac{11}{2}[30]$

$= \frac{330}{2} = 165$

10. $10 + \frac{67}{7} + \frac{64}{7} + \ldots\ldots$

$= \frac{70}{7} + \frac{67}{7} + \frac{64}{7} \ldots\ldots + T_n$

let $T_n < 0$

$\Rightarrow \frac{70}{7} + (n-1).\left(\frac{-3}{7}\right) < 0$

$10 - \frac{3}{7}n + \frac{3}{7} < 0$

$\frac{3n}{7} > \frac{73}{7}$

$3n > 73$

$n > 24.3$ ......

$\because n \in N \Rightarrow n = 25, 26, \ldots\ldots$

$\Rightarrow T_{25}$ = first – ve term

$\therefore$ Sum of positive terms $= S_{24}$

$= \frac{24}{2}\left[2\times10+(24-1)\left(\frac{-3}{7}\right)\right] = 12\times\left[20-\frac{69}{7}\right]$

$= \frac{12\times71}{7} = \frac{852}{7}$

11. $(a_1 + a_5) = 2a_8$

$\Rightarrow 3a_8 = 15$

$\Rightarrow a_8 = 5$

Now $a_2 + a_3 + a_8 + a_{13} + a_{14}$

$= (a_2 + a_{14}) + (a_3 + a_{13}) + a_8$

$= 5a_8 = 25$

12. $a_2 + a_4 + \ldots\ldots\ldots + a_{200} = \alpha$

$a_1 + a_3 + \ldots\ldots\ldots + a_{199} = \beta$

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$d + d + d + \ldots\ldots\ldots + d = \alpha - \beta$

$100d = \alpha - \beta$

$d = \frac{\alpha-\beta}{100}$

13. $T_1 = S_1 = (1 + 2T_1)(1 - T_1)$

$\Rightarrow T_1 = 1/\sqrt{2}\ \ (T_1 > 0)$

Now, $T_1 + T_2 = S_2 = (1 + 2T_2)(1 - T_2)$

$\Rightarrow 2T_2^2 = 1 - T_1 = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$

or $T_2^2 = \frac{\sqrt{2}-1}{2\sqrt{2}}$

14. $n^2 - 1 = 7\left[\frac{10}{2}[10+9.3]\right] = 35.37 \Rightarrow n = 36$

15. $S = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \ldots\ldots\ldots$

General term, $T_n = \frac{1}{n}(1 + 2 + 3 + \ldots\ldots\ldots\ldots\ldots n)$

$T_n = \frac{1}{n}\left\{\frac{n(n+1)}{2}\right\} = \frac{1}{2}(n+1)$

$S_n = \frac{1}{2}\{\Sigma n + \Sigma 1\} = \frac{1}{2}\left\{\frac{n(n+1)}{2} + n\right\}$

$S_{20} = \frac{1}{2}\left[\frac{20\times21}{2} + 20\right] = 115$